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SIMPLIFIED DESIGN EQUATIONS FOR
AN OPTICAL QUARTER-WAVE STACK
REFLECTION FILTER

Robert J. Spry
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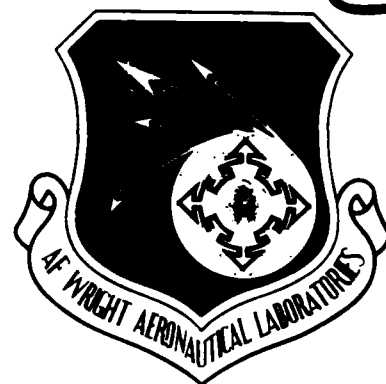
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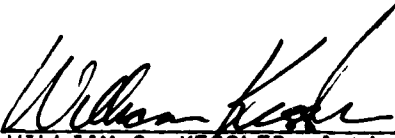
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Formulas are presented which describe the major operational parameters of the high-rejection, narrow bandwidth, quarter-wave stack optical interference filter. Mathematical equivalence is demonstrated between the results of the development of two prominent, present-day theoreticians. To facilitate the understanding of the utility of the simplified equations, a practical design problem has been analyzed. Further areas for application of the equations are suggested.

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FOREWORD

This report describes an in-house study conducted by Robert J. Spry of the Laser Hardened Materials Branch, Electromagnetic Materials Division, Materials Laboratory, Air Force Wright Aeronautical Laboratories, Wright-Patterson Air Force Base, OH 45433 under Project 2422, Task No. 242204, Work Unit 24220401. The work reported herein was performed during the period July 1981 through June 1985 by the author. The report was released on 16 December 1985.

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INTRODUCTION

Optical filters with large absorbance values are much in demand nowadays, largely in relation to the widespread use of moderate or high power lasers. Examples of applications include protection of operators of laser range finders, and removal of the excitation radiation from Raman spectra. Most uses also require that the filter rejection band be narrow, so as to maximize the useful spectral regime. One approach to the solution for these problems is the quarter-wave interference stack filter (Reference 1). The physics of these structures is very mature, having been fully developed and summarized by Professor O. S. Heavens of York University, England (Reference 2). The pertinent features and equations for these filters have also been reviewed by an American expert, Dr. Philip Baumeister (Reference 3).

Many persons familiar with the older theoretical development of Heavens have not immediately recognized the formulas used by Baumeister. The first purpose of this report is to demonstrate the mathematical equivalence between the two schools of the major quarter-wave stack expressions. It is also felt that a summary of the equations in a single place would be handy for use by scientists from other fields who need to make quick estimates of filter performance. A practical example of the application of the equations is also included. Also, it should be noted

that the quarter-wave stack serves as a good first approximation to another popular refractive index spatial variation, that of the sinusoid, or rugate (Reference 4).

DERIVATIONS

1. Physical Description

The quarter-wave stack filter, along with incident and transmitted light rays, is illustrated in Figure 1. The filter is constructed of alternating layers of low-refractive index material, n_L, and high-refractive index material, n_H, deposited upon a substrate of refractive index, n_S. The thicknesses of the low- and high-index materials are d_L, and d_H, respectively. The refractive indices and layer thicknesses are chosen so that the optical thicknesses ($n_i d_i$ products) are one-quarter of the wavelength of light to be rejected. In Figure 1, the case of an even number (2N) of layers is shown. An odd number (2N + 1) of layers is also a practical possibility.

A light ray of wavenumber $\mathcal{V}_0$, wavelength λ_0, and intensity I_0 impinges upon the outer surface of the filter at an angle θ_0 to the surface normal. A beam is reflected about the normal at an equal angle θ_0 with intensity RI_0, where R is the filter reflectance. The transmitted beam exits the back surface of the substrate also at an angle θ_0, with intensity TI_0, T being the transmittance of the filter. In the absence of absorption,

$$R + T = 1 \quad . \tag{1}$$

In Figure 2, a typical reflectance spectrum of a quarter-wave stack interference filter is plotted on a reduced wavenumber scale. The maximum reflectance at the central frequency $\mathcal{V}_0$ is R_{2N}, while the transmittance at $\mathcal{V}_0$ is T_{min}. The bandwidth BW is defined to be the full
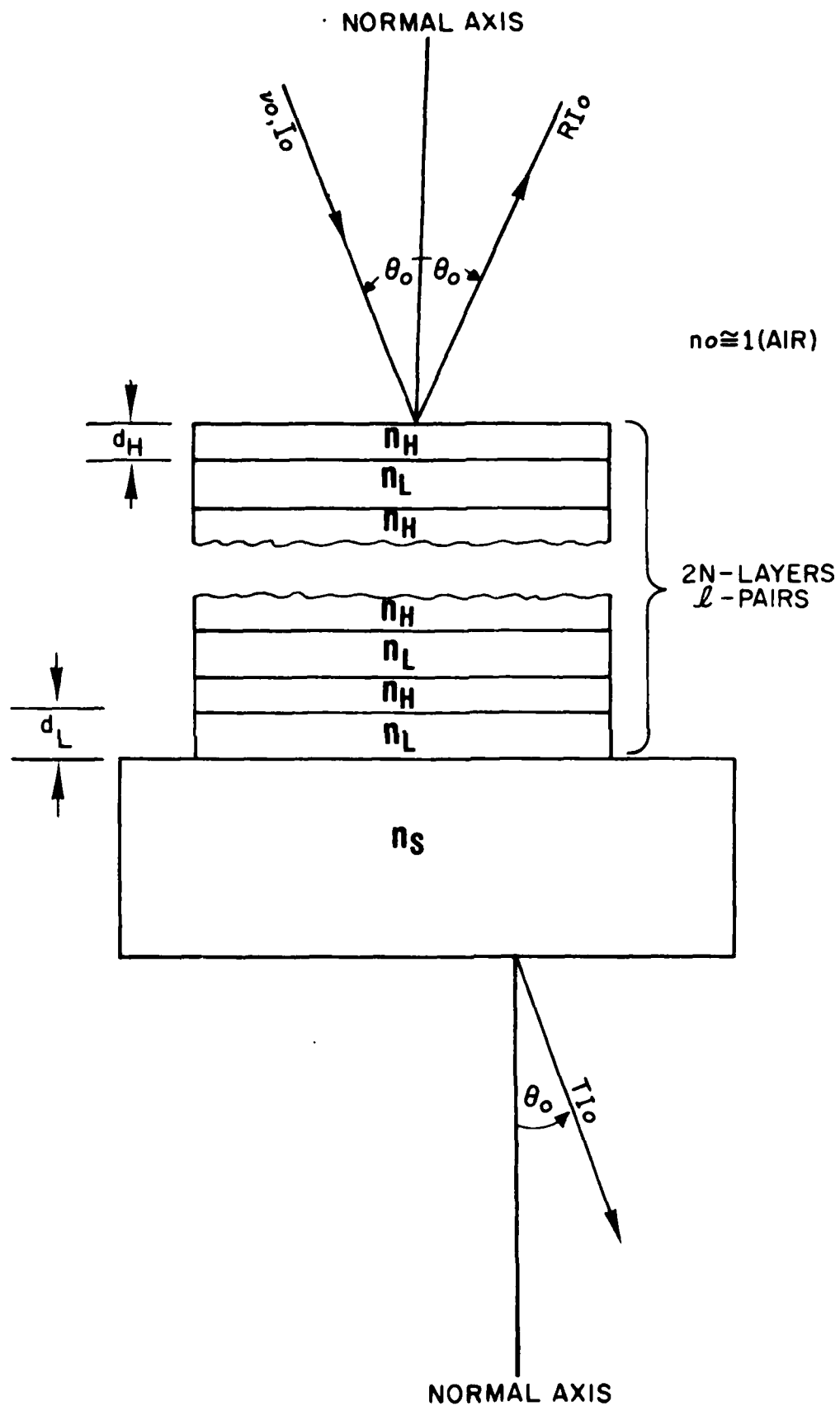
Figure 1. Geometry of the Reflection-Band Filter.
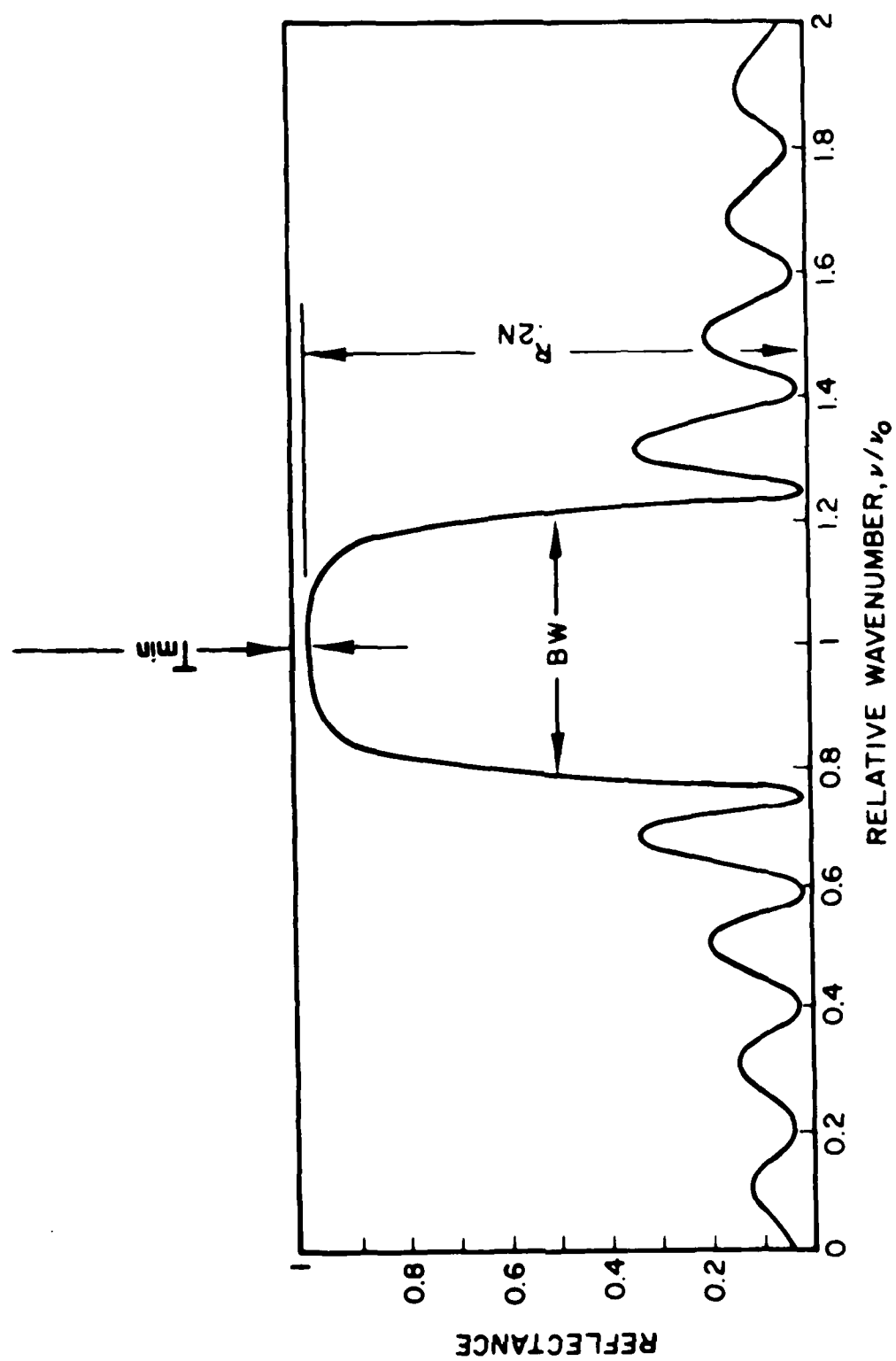
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Figure 2. Spectrum of the Reflection-Band Filter.

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width of the reflectance band (reduced wavenumber scale) at one-half the maximum reflectance. The origin of the smaller relative maxima and minima are beyond the scope of the present discussion.

2. Bandwidth

It can be shown from the product matrix method (References 1 and 2) that

$$BW = \frac{4}{\pi} \arcsin \left(\frac{n_H - n_L}{n_H + n_L} \right) \quad . \tag{2}$$

Making use of the expansion for small x,

$$\arcsin x \cong x \quad , \tag{3}$$

we find

$$BW \cong \frac{4}{\pi} \left(\frac{n_H - n_L}{n_H + n_L} \right) \quad . \tag{4}$$

If we further define

$$\Delta n \equiv \left(n_H - n_L \right) \quad , \tag{5}$$

and

$$\bar{n} \equiv \left(\frac{n_H + n_L}{2} \right) \quad , \tag{6}$$

we arrive at

$$BW \cong \left(\frac{2}{\pi} \frac{\Delta n}{\bar{n}} \right) \quad , \tag{7}$$

the result of Baumeister.

3. Reflectance and Transmittance

For an even number of layer pairs, Heavens (Reference 1) has derived

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the reflectance maximum as

$$R_{2N} = \left(\frac{n_{sf} - n_o}{n_{sf} + n_o}\right)^2 \quad , \tag{8}$$

and for an odd number of pairs as

$$R_{2N+1} = \left(\frac{fn_H^2 - n_o n_s}{fn_H^2 + n_o n_s}\right) \quad . \tag{9}$$

Here, f is the standing wave ratio

$$f = \left(\frac{n_H}{n_L}\right)^{2N} \quad . \tag{10}$$

If we let

$$y = \frac{n_o}{n_{sf}} \quad , \tag{11}$$

then Eq. (8) becomes

$$R_{2N} = \left(\frac{1 - y}{1 + y}\right)^2 \quad . \tag{12}$$

For the desired case of large reflectance values

$$y \ll 1 \quad , \tag{13}$$

so that R_{2N} may be expanded as

$$R_{2N} = (1 - 2y + y^2)(1 - y + y^2 \text{----})^2 \quad , \tag{14}$$

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or

$$R_{2N} \simeq \left(1 - 4y\right) \quad . \tag{15}$$

Thus,

$$T_{min} \simeq 4y = \frac{4n_o/n_s}{f} \quad . \tag{16}$$

If the antireflection coatings have already been applied to the front and back surfaces of the quarter-wave stack to produce zero reflection at the boundaries, and if we let ℓ = 2N and V = f, we obtain

$$V = \left(\frac{n_H}{n_L}\right)^{\ell} \quad , \tag{17}$$

and

$$T_{min} \simeq \frac{4}{V} = 4 \left(\frac{n_H}{n_L}\right)^{-\ell} \quad , \tag{18}$$

bringing the previous results into agreement with the expressions of Baumeister. These antireflection coatings also have a quarter-wave optical thickness and refractive index values given by

$$n_a = \sqrt{\bar{n} \, n_o} \quad , \tag{19}$$

and

$$n_b = \sqrt{\bar{n} \, n_s} \quad . \tag{20}$$

When actually designing a filter, one is often interested in the number of layers to be deposited in achieving a given level of performance. To find this required number of layer pairs, we first compute the natural logarithm of both sides of Eq. (18):

$$\ln\left(\frac{T_{min}}{4}\right) = -\ell \ln\left(\frac{n_H}{n_L}\right) \quad . \tag{21}$$

Further, utilizing Eq. (5) and the series expansion

$$\ln(1 + z) = (z - \frac{z^2}{2} + ---) \quad , \tag{22}$$

Eq. (21) simplifies to:

$$\ell \simeq \left(\frac{n_L}{\Delta n}\right) \ln\left(\frac{4}{T_{min}}\right) \quad . \tag{23}$$

For most practical cases n_L is approximately equal to $\bar{n}$. Thus, combining Eq. (7) and Eq. (23) yields

$$\ell \simeq \frac{2}{\pi(BW)} \ln\left(\frac{4}{T_{min}}\right) \quad . \tag{24}$$

This is a most practical equation for the designer, because it clearly relates the trade-offs among bandwidth, optical rejection, and number of required layer pairs.

In our present model we have assumed that the absorption within the filter is identically zero. It is nevertheless useful to calculate an

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effective peak absorbance, defined in the usual manner in terms of the real transmittance:

$$D_{max} \cong - \log (T_{min}) \quad . \tag{25}$$

Inserting Eq. (18) into Eq. (25) gives

$$D_{max} \cong \log V - 0.602 \quad , \tag{26}$$

while utilizing Eq. (22) and Eq. (17) produces

$$D_{max} \simeq \ell \left(\log e \right) \left(\frac{\Delta n}{n_L} \right) - 0.602 \quad . \tag{27}$$

Finally, with the help of Eq. (7) and the previous approximation for n_L, we obtain Baumeister's result,

$$D_{max} \simeq 0.434 \frac{\pi}{2} (BW) \ell - 0.602 \quad . \tag{28}$$

This formula is similar to Eq. (24) in its usefulness in design applications.

4. Angle Shift of the Reflection Band Maximum

For the case of a single layer of thickness d and refractive index n, the condition for a reflectance maximum is:

$$\frac{2nd \cos \theta}{\lambda_0} = k + 1/2 \quad , \tag{29}$$

where θ is the angle of refraction inside the medium and k is an integer (Reference 5). Setting k = o, we will combine Eq. (29) with the result from Snell's law,

$$n \sin \theta = n_0 \sin \theta_0 \quad , \tag{30}$$

to give the fractional wavelength shift

$$\frac{\delta \lambda_0}{\lambda_0} = \sqrt{1 - \left(\frac{n_0}{n} \sin \theta_0\right)^2} - 1 \quad . \tag{31}$$

Turning to our quarter-wave stack, we next replace n by $\bar{n}$, and expand the above radical for small angles (in radians) to obtain

$$\frac{\delta \lambda_0}{\lambda_0} \simeq -\frac{1}{2}\left(\frac{n_0 \theta_0}{\bar{n}}\right)^2 \quad . \tag{32}$$

Using $\bar{n}$ here is not completely accurate, but the correct, effective index does lie between n_H and n_L. Eq. (32) is the same result obtained by Baumeister, except that his text seems to be in error because it refers to the angle in Eq. (32) as "the mean refracted angle inside the periodic stack."

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We usually will only be able to tolerate a certain small wavelength shift, which we will define as a fraction Υ of the bandwidth:

$$\left(\frac{\delta\lambda_o}{\lambda_o}\right)_{max} \cong \Upsilon \, BW \qquad . \tag{33}$$

Combining Eqs. (7), (32), and (33) yields for the maximum tolerable angle of incidence,

$$\theta_{max} = 2\sqrt{\frac{\Upsilon}{\pi} \Delta n \, \bar{n}} \qquad . \tag{34}$$

SECTION III

DESIGN EXAMPLE

We will next apply our mathematical results to a practical example recently considered at the Materials Laboratory (Reference 6). We wanted to produce a high-reflectance narrow band filter for 5300 Å, with minimum angle shift. A major concern was the number of required layer pairs, since the actual manufacture for a large number would be prohibitive. Further, n_L was constrained to the value 2.37, while n_H could be only about 6 percent greater.

The required minimum transmittance is plotted in Fig. 3 using Eq. (18). Practical required transmittance values were in the 10^{-2} to 10^{-4} range. A family of equations was generated by allowing n_H to assume values between 2.39 and 2.52. Alternatively, Eq. (23) was used to generate similar design considerations in Fig. 4. It is clear from both graphs that there is a severe trade-off among T_{min}, ℓ, and Δn. To reduce ℓ to a practical level, it is thus required to make Δn reasonably large and reduce expectations for T_{min}. For example, $T_{min} = 10^{-2}$ and $n_H \geq 2.47$ would keep $\ell < 150$.
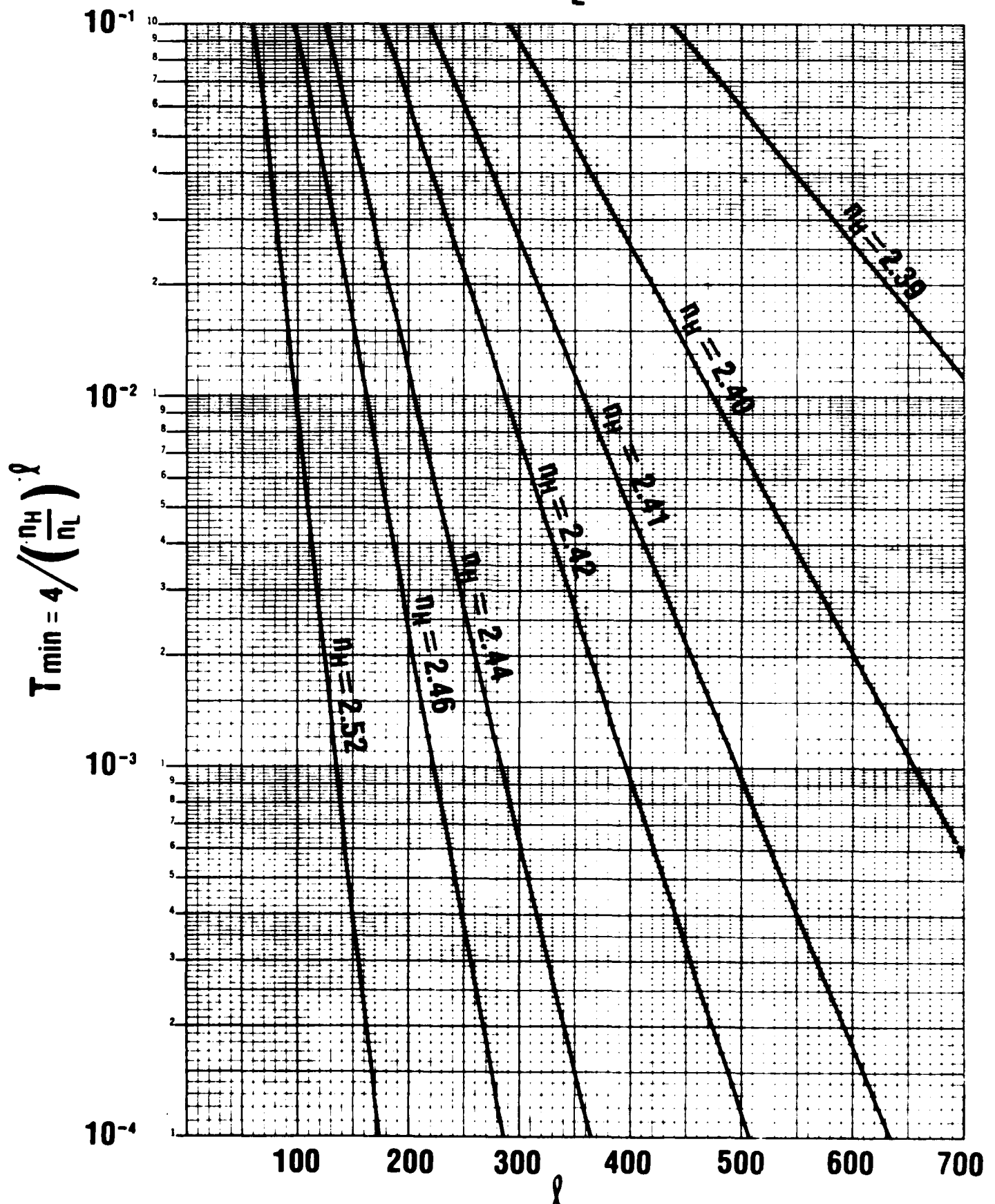
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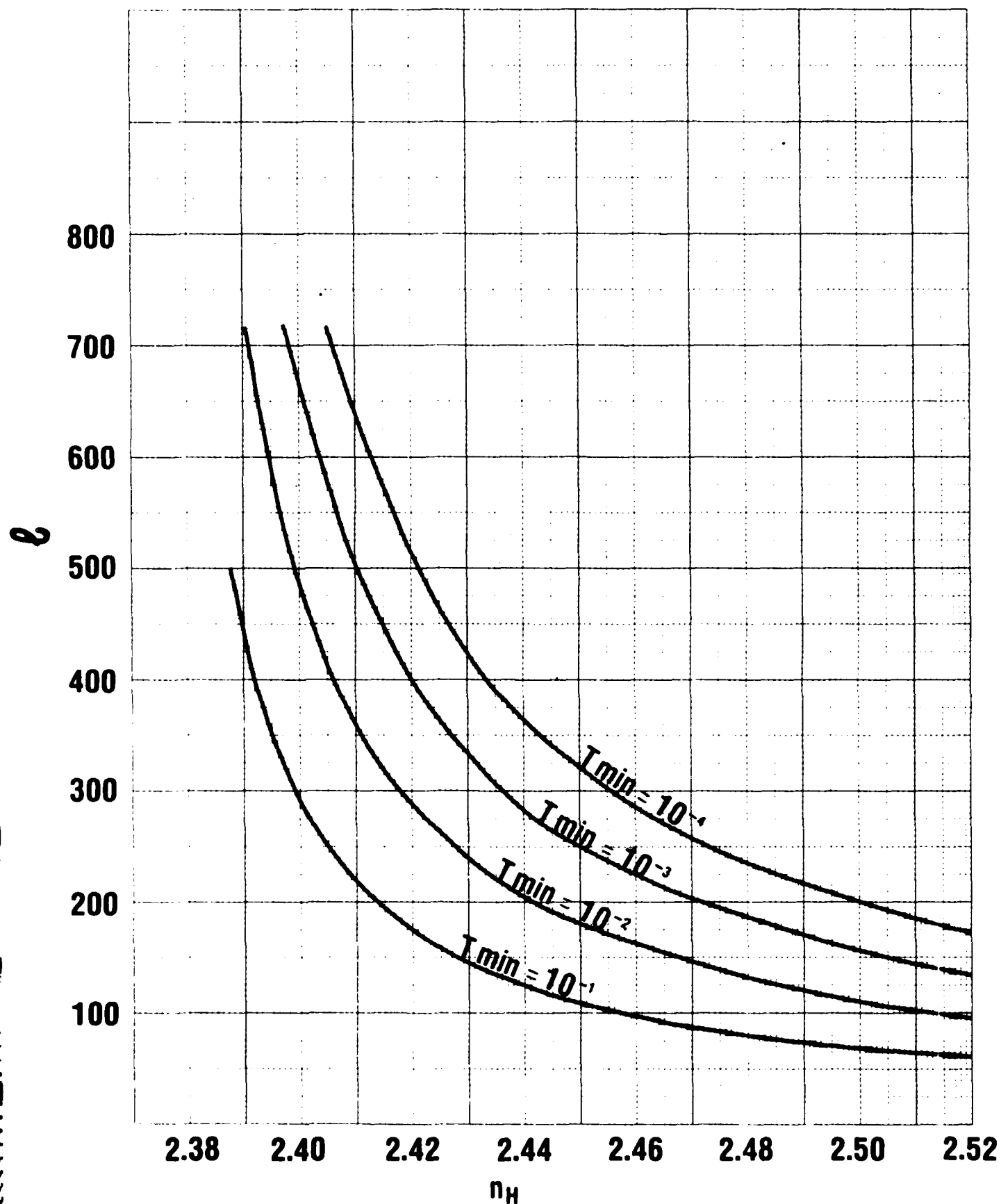Figure 3.   Minimum Transmission vs. Number of Layer Pairs.

Figure 4. Number of Layer Pairs vs. the High Refractive Index.

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Next, we consider the bandwidth, governed by the parameters of Eq. (7) as shown in Fig. 5. Even up to the maximum allowed value of 2.52 for n_H, the bandwidth is less than 0.04, a quite acceptable value. Thus, the bandwidth does not impose any limitations upon the design or manufacture of our case study filter.

The angle shift of the peak reflectance is the only design problem with which we must still contend. To facilitate our choices, we have plotted Eq. (34) in Fig. 6 using the same family of values for n_H as was used in Fig. 3. We wish the required wavelength peak shift to be no more than 20 percent of the bandwidth. We previously found that n_H must be ≥ 2.47. From the curves we find $\theta_{max} \cong 15°$, a good value for the angle of incidence, or 30° for the field of view (F.O.V.). This is quite compatible with many practical optical systems.
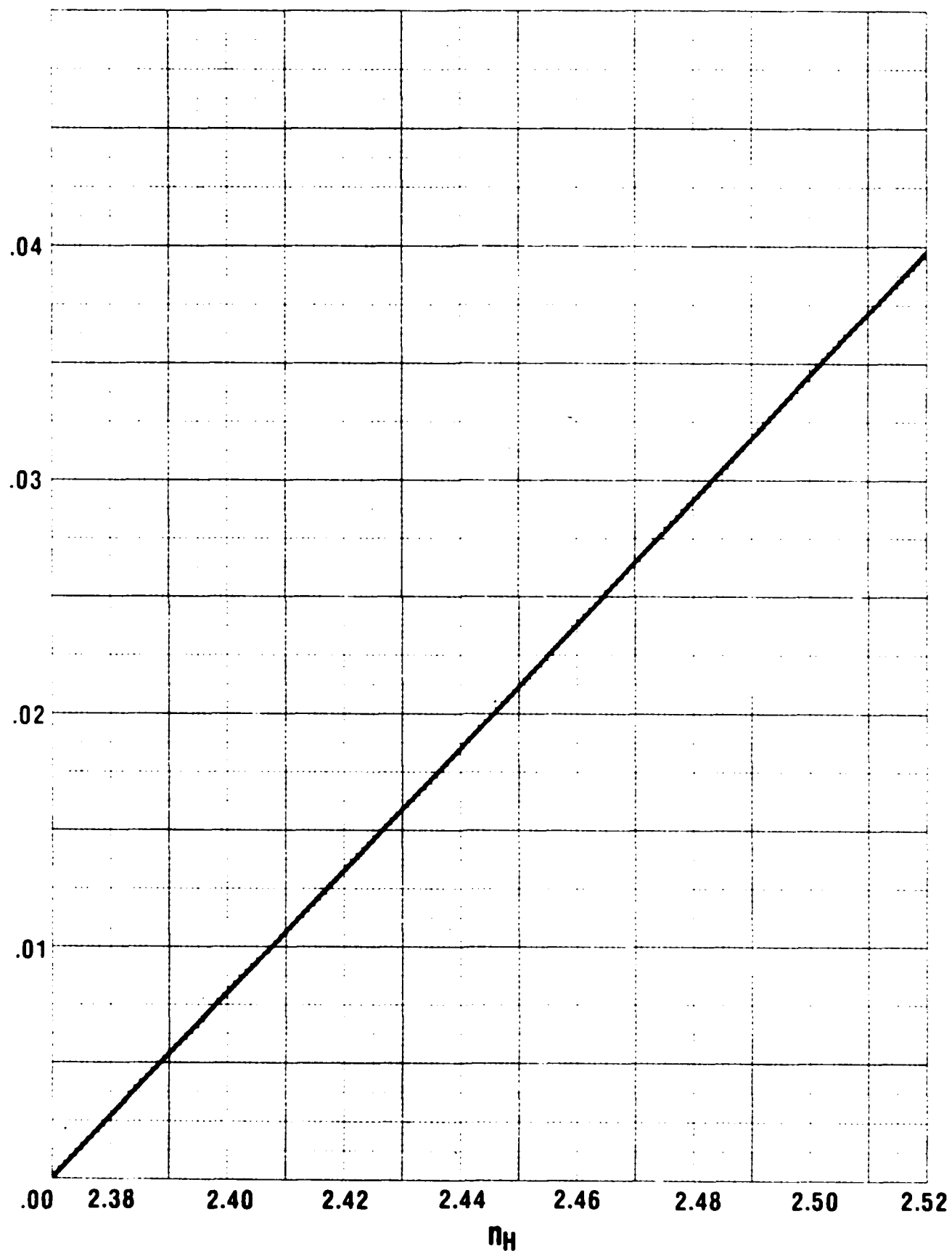
Figure 5.   Width of the Reflection Band vs. the High Refractive Index.
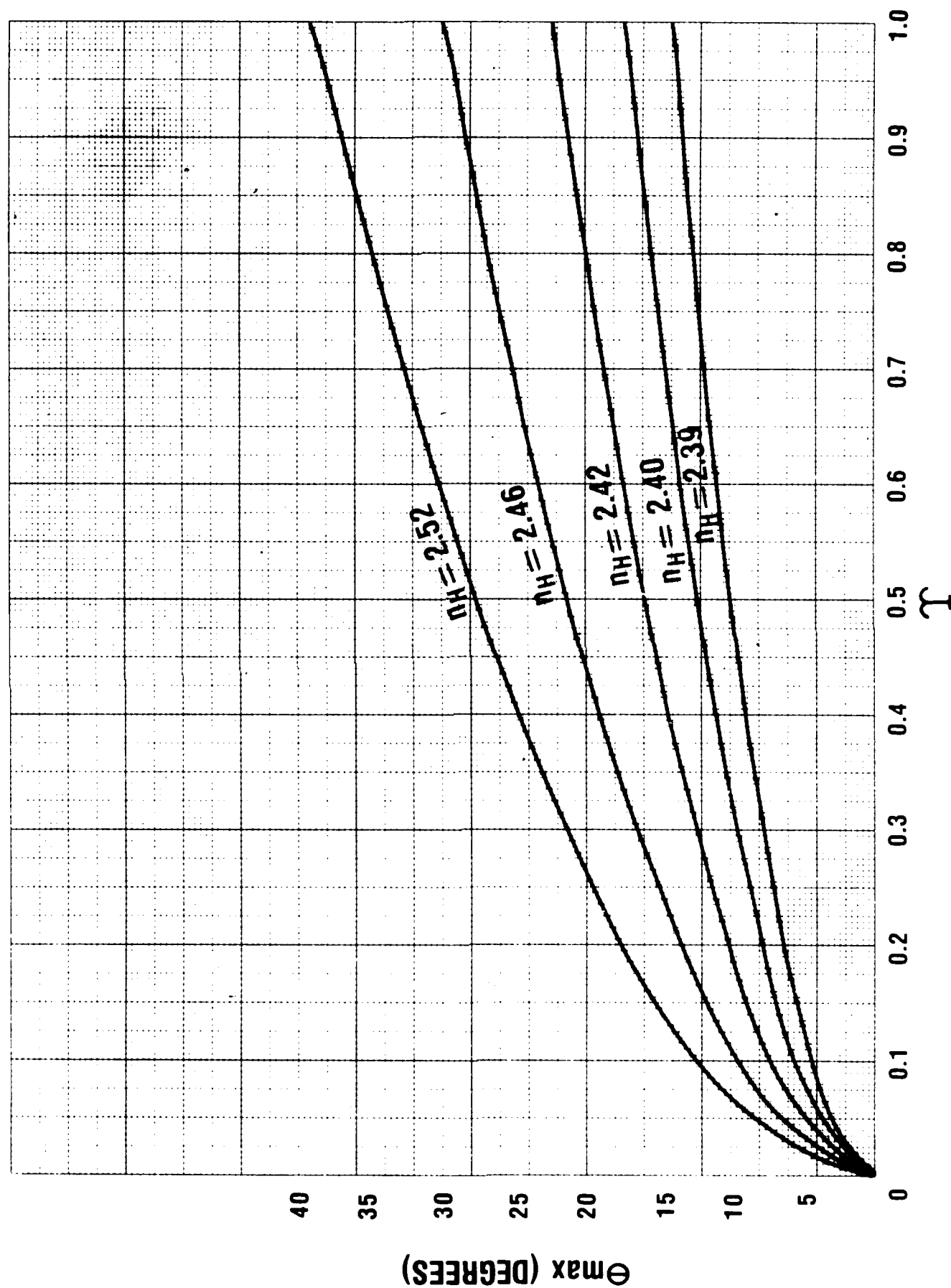
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Θmax vs. Υ

Figure 6. Maximum Angle of Incidence vs. Maximum Tolerable Fractional Spectral Shift.

SECTION IV

CONCLUSIONS

We have collected or derived the equations describing the major operational parameters for high-rejection, narrow bandwidth, quarter-wave stack interference filters. In doing so we have demonstrated the mathematical equivalence between expressions from the two major theoretical developments used by most scientists at the present time. We have presented simplified design equations are presented for the transmittance minimum, reflectance maximum, absorbance maximum, bandwidth, required number of layer pairs, and angle shift of the reflectance maximum.

This report should be useful to the novice who wishes to obtain quick, moderately accurate results without indulging in the intricacies of matrix manipulations and tedious computer calculations. This category may include persons whose major efforts are concerned with materials, lasers, spectroscopy, nonlinear optics, systems, and management. We also feel that this collection of equations helps one maintain cognizance of the pertinent physics, something often lost when encumbered by numerical methods. To demonstrate the utility of the equations contained herein, we have carefully analyzed a practical design problem. Finally, we wish to emphasize that through the connection via Fourier's theorem, the formulas for the quarter-wave stack filter serve as a quick, first approximation for calculating the major features of filters having sinusoidal refractive index profiles.

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